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Neutrino - electron processes in a strong magnetic field and plasma

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Abstract

The total set of the neutrino - electron processes in a presence of both components of an active medium, hot dense plasma and strong magnetic field, is investigated for the first time. The contribution of the processes $\nu e^- \to \nu e^-$, $\nu e^+ \to \nu e^+$, $\nu \to \nu e^- e^+$, $\nu e^- e^+ \to \nu$ is shown to dominate and not to depend on the chemical potential of electron-positron gas. Relatively simple expressions for the probability and mean losses of the neutrino energy and momentum are obtained, which are suitable for a quantitative analysis.

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An understanding of the important role of neutrino interactions in astrophysical processes stimulates a constantly growing interest in the neutrino physics in a dense medium [1]. For example, in a collapsing stellar core where a large number of neutrinos is produced, the density amounts up to the value of a nuclear density, and matter becomes opaque to neutrinos. The elastic scattering on nuclei was usually considered as the main source of neutrino opacity. A contribution of the neutrino-electron scattering $\nu e^- \to \nu e^-$ was estimated to be essentially smaller, and it was not included in the earlier attempts of collapse simulation (see, e.g., [2] and references therein). However, as the analysis showed [3], neutrino-electron scattering can contribute essentially to the energy balance of the collapsing stellar core. A numerical computation of the differential probability of the reaction $\nu e^- \to \nu e^-$ in degenerate electron plasma was performed in a paper [4], where an influence of relatively small magnetic fields was taken into account.

It should be noted that a consideration of a strong magnetic field as a medium, along with a dense matter, is physically justified indeed. Really, the field strengths inside the astrophysical objects can reach the critical Schwinger value $B_e = m_e^2/e \simeq 4.41 \cdot 10^{13}$ G [1] , and even exceed it essentially. In the present view, very strong magnetic fields at a level of 10^{16}G can exist inside the astrophysical cataclysms like a supernova explosion or a coalescence of neutron stars. For example, such fields could be generated in a supernova envelope due to the mechanism suggested by Bisnovatyi-Kogan [5]. It should be emphasized that such field is really rather dense medium with the mass density

$$\rho = \frac{B^2}{8\pi} \simeq 0.4 \cdot 10^{10} \frac{\mathrm{g}}{\mathrm{cm}^3} \cdot \left(\frac{B}{10^{16}\mathrm{G}} \right)^2 , \tag{1}$$

which is comparable with the plasma mass density $10^{10} - 10^{12}\mathrm{g/cm}^3$ to be typical for the envelope of an exploding supernova. It is known that such intense fields make an active influence on quantum processes, thus allowing the transitions which are kinematically forbidden in vacuum.

In this paper we investigate the total set of the neutrino - electron processes in a presence of both components of an active medium, hot dense plasma and strong magnetic field. This set includes not only the "canonical" scattering $\nu e^- \to \nu e^-$, $\nu e^+ \to \nu e^+$, $e^+ e^- \leftrightarrow \nu \bar\nu$, but the "exotic" process

[1] We use natural units in which $c = \hbar = 1$.

of the synchrotron emission of the neutrino pair and the reverse process, $e \leftrightarrow e\nu\bar{\nu}$, and the processes of production and absorption of the electron-positron pair $\nu \to \nu e^- e^+$, $\nu e^- e^+ \to \nu$ as well. The "exotic" processes become opened only in the presence of a magnetic field.

We consider the physical situation when the field strength B appears to be the largest physical parameter, while the electron mass is the smallest one

$$eB \gg E^2, \, \mu^2, \, T^2 \gg m_e^2, \tag{2}$$

where E is the neutrino energy, μ is the chemical potential of electrons, T is the temperature of plasma. In this limit, the electrons and the positrons occupy the lowest Landau level.

We use the effective Lagrangian of the neutrino - electron interaction in the local limit

$$\mathcal{L} = \frac{G_F}{\sqrt{2}} [\bar{e} \gamma_\alpha (g_V - g_A \gamma_5) e] \, [\bar{\nu} \gamma^\alpha (1 - \gamma_5) \nu] \, , \tag{3}$$

where $g_V = \pm 1/2 + 2 sin^2 \theta_W$, $g_A = \pm 1/2$. Here the upper signs correspond to the electron neutrino ($\nu = \nu_e$) when both Z and W boson exchange takes part in a process. The lower signs correspond to μ and τ neutrinos ($\nu = \nu_\mu, \nu_\tau$), when the Z boson exchange is only presented in the Lagrangian (3).

An amplitude of any above-mentioned neutrino - electron process could be immediately obtained from the Lagrangian (3) where the known solutions of the Dirac equation in a magnetic field should be used. At first glance all the neutrino - electron processes should be strongly suppressed in the considered limit (2), because the amplitude tends to zero in the limit of massless electron. It can be best demonstrated for the processes $e^+ e^- \leftrightarrow \nu\bar{\nu}$. Really, the total spin of the neutrino - antineutrino pair in the center-of-mass system is equal to 1, while the total spin of the electron - positron pair on the lowest Landau level is equal to zero. Thus the process amplitude is strictly zero for the case of the local interaction and massless particles, and it is suppressed in the considered relativistic limit.

However, as the analysis shows, all the neutrino - electron processes described by the Lagrangian (3) can be separated into two parts:

i) the processes with the neutrino - antineutrino pair in the initial or in the final state, $e^- e^+ \leftrightarrow \nu\bar{\nu}$, $e \leftrightarrow e\nu\bar{\nu}$, where the above-mentioned suppression remains valid after integration over the phase space;

ii) the processes where the neutrino presents both in the initial and in the final state, $\nu e^{\mp} \to \nu e^{\mp}$, $\nu \leftrightarrow \nu e^- e^+$, and the similar processes with the antineutrino, where a non-trivial kinematic enhancement takes place in the integration over the phase space, which provides the exact compensation of the amplitude suppression.

We investigate first the "canonical" process of neutrino scattering on the electrons of magnetized plasma. The probability of this process in a unit time has a physical meaning only being integrated over both the final and the initial electron states as well

$$W(\nu e^- \to \nu e^-) = \frac{1}{\mathcal{T}} \int |\mathcal{S}|^2 \, d\Gamma_{e^-} \, f_{e^-} \, d\Gamma'_{e^-} \, (1 - f'_{e^-}) \, d\Gamma'_\nu \, (1 - f'_\nu), \tag{4}$$

where $\mathcal{T}$ is the total interaction time, $\mathcal{S}$ is the matrix element of the transition, $d\Gamma$ is the phase-space element of a particle, f is its distribution function, $f'_\nu = [exp((E' - \mu_\nu)/T_\nu) + 1]^{-1}$, μ_ν, T_ν are the chemical potential and the temperature of the neutrino gas, $f_{e^\mp} = [exp((E_\mp \mp \mu)/T) + 1]^{-1}$, μ, T are the chemical potential and the temperature of the electron-positron gas. We do not present here the details of integration over the phase space of particles, which will be published in an extended paper. The result of our calculation of the probability (4) can be presented in the form

$$\begin{aligned}
W(\nu e^- \to \nu e^-) &= \frac{G_F^2 e B T^2 E}{4\pi^3} \\
&\times \Bigg\{ (g_V + g_A)^2 (1-u)^2 \int\limits_0^{x\tau\frac{1+u}{2}} \frac{d\xi}{(1 - e^{-\xi})(1 + e^{-x + \eta_\nu + \xi/\tau})} \ln \frac{1 + e^\eta}{1 + e^{-\xi + \eta}} \\
&+ (g_V - g_A)^2 (1+u)^2 \int\limits_0^{x\tau\frac{1-u}{2}} \frac{d\xi}{(1 - e^{-\xi})(1 + e^{-x + \eta_\nu + \xi/\tau})} \ln \frac{1 + e^\eta}{1 + e^{-\xi + \eta}} \\
&+ [(g_V + g_A)^2 (1-u)^2 + (g_V - g_A)^2 (1+u)^2] \\
&\times \int\limits_0^{\infty} \frac{d\xi}{(e^\xi - 1)(1 + e^{-x + \eta_\nu - \xi/\tau})} \ln \frac{1 + e^\eta}{1 + e^{-\xi + \eta}} \Bigg\},
\end{aligned} \tag{5}$$

where $x = E/T_\nu$, $\tau = T_\nu/T$, $\eta_\nu = \mu_\nu/T_\nu$, $\eta = \mu/T$, $u = \cos\theta$, θ is an angle between the initial neutrino momentum $\mathbf{p}$ and the magnetic field induction $\mathbf{B}$. The dependence of the scattering probability (5) on the electron density,

$n = n_{e^-} - n_{e^+}$, is defined by the dependence on the chemical potential of electrons

$$\mu = \frac{2\pi^2 n}{eB}.$$

Let us note that the expression (5) is valid for the cases of both hot ($\mu \ll T$) and cold ($\mu \gg T$) plasma. The probability of the neutrino scattering on positrons, $\nu e^+ \to \nu e^+$, can be obtained from Eq. (5) by a simple change of sign of the electron chemical potential, $\mu \to -\mu$.

The probabilities of the remaining "exotic" processes $\nu \leftrightarrow \nu e^- e^+$ are defined similarly to (4) with the substitution $f_{e^-} \to (1 - f_{e^+})$ in every transposition of the electron from the initial (final) state to the final (initial) one. The results of our calculations are

$$W(\nu \to \nu e^- e^+) = \frac{G_F^2 eBT^2 E}{4\pi^3} \tag{6}$$
$$\times \Bigg\{ (g_V + g_A)^2 (1-u)^2 \int\limits_0^{x\tau\frac{1+u}{2}} \frac{d\xi}{(1-e^{-\xi})(1+e^{-x+\eta_\nu+\xi/\tau})} \ln \frac{\cosh\xi + \cosh\eta}{1+\cosh\eta}$$
$$+ (g_V - g_A)^2 (1+u)^2 \int\limits_0^{x\tau\frac{1-u}{2}} \frac{d\xi}{(1-e^{-\xi})(1+e^{-x+\eta_\nu+\xi/\tau})} \ln \frac{\cosh\xi + \cosh\eta}{1+\cosh\eta} \Bigg\},$$

$$W(\nu e^- e^+ \to \nu) = \frac{G_F^2 eBT^2 E}{4\pi^3} [(g_V + g_A)^2 (1-u)^2 + (g_V - g_A)^2 (1+u)^2]$$
$$\times \int\limits_0^\infty \frac{d\xi}{(e^\xi - 1)(1+e^{-x+\eta_\nu-\xi/\tau})} \ln \frac{\cosh\xi + \cosh\eta}{1+\cosh\eta}. \tag{7}$$

We note that rather the total probability of the neutrino interaction with magnetized electron-positron fraction of plasma

$$\begin{aligned} W(\nu \to \nu) &= W(\nu \to \nu e^- e^+) + W(\nu e^- e^+ \to \nu) + \\ &+ W(\nu e^- \to \nu e^-) + W(\nu e^+ \to \nu e^+) \end{aligned} \tag{8}$$

has a physical meaning, than the probabilities (5), (6), (7) separately. For the total probability we obtain a more simple expression

$$
\begin{aligned}
W(\nu \to \nu) \;&=\; \frac{G_F^2 eBT^2 E}{4\pi^3} \Big\{ (g_V + g_A)^2 (1-u)^2 \left[F_1 \left(\frac{x\tau(1+u)}{2} \right) - F_1(-\infty) \right] \\
&+\; (g_A \to -g_A;\; u \to -u) \Big\}, \qquad (9)
\end{aligned}
$$

where

$$
F_k(z) = \int\limits_0^z \frac{\xi^k d\xi}{(1 - e^{-\xi})(1 + e^{-x + \eta_\nu + \xi/\tau})}. \qquad (10)
$$

Both the probability (6) and (9) do reproduce our result [6, 7] obtained for the case of a pure magnetic field, in the limit of a rarefied plasma (T, μ, T_ν, $\mu_\nu \to 0$).

It is interesting to note that the dependence on the electron chemical potential magically cancelled in the total probability (9), whereas each of the partial probabilities (5), (6), (7) does depend on μ. It means that the total probability does not depend on the electron density n in the strong field limit (2). We do not know a physical underlying reason of this independence up to now. Probably, some property of a completeness of the considered set of processes with respect to the electrons manifests itself here.

The probability (9) defines the partial contribution of the considered processes into the neutrino opacity of the medium. The estimation of the neutrino mean free path with respect to the neutrino-electron processes yields

$$
\lambda_e = \frac{1}{W} \simeq 170 \text{ km} \cdot \left(\frac{10^3 B_e}{B} \right) \left(\frac{5 \text{ MeV}}{T} \right)^3. \qquad (11)
$$

It should be compared with the mean free path caused by the interaction with nuclei, which is evaluated to be of order of 1 km at the density value $\rho \sim 10^{12}$ g/cm^3. At first glance the influence of the neutrino-electron reactions on the process of neutrino propagation is negligibly small. However, a mean free path does not exhaust the neutrino physics in a medium. The mean values of the neutrino energy and momentum loss are also essential in astrophysical applications.

The mean values of the neutrino energy and momentum loss [2] could be defined by the four-vector

$$Q^{\alpha} = - E \left(\frac{dE}{dt}, \frac{d\mathbf{p}}{dt} \right) = E \int q^{\alpha} dW, \tag{12}$$

where E and $\mathbf{p}$ are the neutrino energy and momentum, q is the difference of the momenta of the initial and final neutrinos, $q = p - p'$, dW is the total differential probability of the processes considered. The zero component Q^0 is connected with the mean neutrino energy loss in a unit time, the space components $\mathbf{Q}$ are connected similarly with the neutrino momentum loss in a unit time.

The four-vector Q^{α} was calculated in our papers [6, 7] for the case of a pure magnetic field. The losses are connected in this case with the pair production by a neutrino propagating in a strong magnetic field, $\nu \to \nu e^- e^+$, which is the only possible process in the absence of plasma.

The result of our calculation of the zeroth and third components (the magnetic field is directed along the third axis) of the four-vector Q^{α} in a strongly magnetized plasma is

$$\begin{aligned} Q_{0,3} &= \frac{G_F^2 eBT^3E^2}{4\pi^3} \Big\{ (g_V + g_A)^2 (1-u)^2 \left[F_2 \left(\frac{x\tau(1+u)}{2} \right) - F_2(-\infty) \right] \\ &\pm (g_A \to -g_A;\ u \to -u) \Big\}, \end{aligned} \tag{13}$$

where the function $F_2(z)$ is defined in Eq. (10), the plus or minus signs correspond to the zeroth and third components. Our result for the four-vector of losses obtained in the case of a pure magnetic field [6, 7], is reproduced from Eq. (13) in the limit of a rarefied plasma (T, T_ν, $\mu_\nu \to 0$).

Finally, we have investigated the neutrino-electron interactions in strongly magnetized plasma, taking into account the total set of neutrino - electron processes, both "canonical", $\nu e^{\mp} \to \nu e^{\mp}$, and "exotic", $\nu \leftrightarrow \nu e^- e^+$. The total probability and the four-vector of the mean values of the neutrino energy and momentum loss have been calculated. The surprising result is that these values appear not to depend on the electron density.

There is good reason to believe that the results obtained could be useful in astrophysical applications.

[2] In general, a neutrino can both loose and absorb the energy and momentum.

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